

Problem #4

SDMo ID #797

You can use complementary counting and can first find the number of ways where the sum of the smallest and biggest number is less than 15. If you have the smallest and biggest numbers, you can find how many numbers the middle one can be to find the total number of combinations.

First and last number	How many numbers the middle can be	
1 - 3	1	} $1+2+3+\dots+11$
1 - 4	2	
1 - 5	3	
⋮	⋮	
1 - 13	11	
2 - 4	1	} $1+2+\dots+9$
2 - 5	2	
⋮	⋮	
2 - 12	9	
⋮	⋮	
6 - 8	1	} 1

where the smallest and biggest numbers sum to be less than 15

The total number of combinations is $1+2+3+\dots+11 + 1+2+3+\dots+9 + 1+2+3+\dots+7 + 1+2+3+\dots+5 + 1+2+3+1$

$$\begin{aligned}
 &= \frac{11+26}{2} + \frac{9+5}{2} + \frac{7+8}{2} + \frac{5+6}{2} + \frac{3+4}{2} + 1 \\
 &= 66 + 45 + 28 + 15 + 6 + 1 \\
 &= 161
 \end{aligned}$$

The probability that sum of the smallest and biggest number is at least 15 is

$$1 - \frac{161}{\binom{14}{3}} = 1 - \frac{7 \cdot 23}{2 \cdot 4 \cdot 26} = 1 - \frac{21}{52} = \boxed{\frac{29}{52}}$$